A CREDIT RISK MODEL FOR BANK’S LOAN PORTFOLIO & OPTIMIZE THE VAR

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Abstract
The New Basel accord has highlighted the need for models of the credit risk in portfolios of corporate loans. There are really no such models of the risks in corporate loan portfolios even though there is a well established industry – credit scoring – in modelling the risk of individual loans. This paper proposes a method to calculate portfolio credit risk. Individual default risk estimates are used to compose a value-at-risk (VaR) measure for credit risk. This empirical research study attempts to measure credit risk of a bank’s corporate loan portfolio, including firms from 5 different business sectors. Credit loss distributions for each segment are selected and used in a Monte Carlo simulation process to generate the loss distribution of the portfolios. We use samples of Agriculture, Manufacturing and mining, Construction and housing, Exports, Trade services and miscellaneous to obtain efficiency estimates for individual firms in each industry. The monthly observations of the total amount of corporate loans and defaults across various sectors were supplied by a major commercial bank in Iran. This period covers 36 monthly observations. The observed sectoral default rates are needed to be simulated to obtain a nicely shaped distribution. Monte Carlo simulation is applied for 1,500 times. Based on the simulated default rates, the expected sectoral default rates are computed. The sectoral weights in the whole loan portfolio are multiplied by the expected sectoral default rates matrix, considering cross-sectoral correlations to get the total amount of the bank’s credit risk and capital requirement. Finally, we model credit risk of portfolio to optimize the percentage of loan allocation to each cluster in order to minimize the VaR of the portfolio.

Keywords: IRB approach, Risk management, Default rate, Credit risk, Value at Risk

1. Introduction

The growth in liquidity of credit markets and the active management of credit risk are among the most significant developments in commercial banking in recent years. These developments hold the potential to permanently reduce the risk profile and improve the financial performance of commercial banks.

Almost all banking activities inherently incur credit risk. So, accurate measurement of credit risks banks bear are critical since the Basel standards I,II and III requires all banks to allocate sufficient amount of capital for all risks they incur through their banking activities. On the
other hand, the newly proposed Basel Accord III encourages banks to use either foundation or advanced internal ratings base (IRB) approach in measuring credit risks of their portfolios. The use of technical and advanced approaches in the measurement of credit risk of banks’ portfolios has nowadays become a very hot issue. The most recent technical report issued by the Basel Committee has concentrated heavily on the measurement of credit risk. Using either foundation or advanced Internal Ratings Base (IRB) approaches. The foundation IRB approach employs predetermined parameters of “exposure at default” and “loss given default,” and requires the estimation of bank specific default rates. On the other hand, advanced IRB approach requires that all underlying parameters are estimated by individual banks. The application of IRB approaches needs a large scale of detailed and classified historical observations. IRB models may be easily applied by simulating historical observations a sufficient number of times to obtain nicely shaped normal distributions. Hence, the simulated sectoral default distributions can be used to determine credit risk of a bank’s loan portfolio and amount of capital requirement (Teker, Akcay, & Turan, 2006).

While credit portfolio management was created for a defensive purpose (to reduce the losses in the loan portfolio), the rational management of economic capital is driving this function to embrace return and risk in its performance objectives. The continued progress toward rational asset management will require commercial banks to overcome formidable obstacles that impede or prevent them from fully integrating their activities into the market. Models of portfolio credit risk predict the distribution of credit loss or value for a portfolio. The loss or value distribution can be used for many applications in risk management such as Economic Capital and Value-at-Risk assessment, performance evaluation and marginal risk estimation.

The quantitative loan portfolio management process solutions facilitate the decrease in portfolio risk through better risk identification and risk diversification, and increase portfolio profitability through the reduction of portfolio volatility and the increase in customer profitability.

Credit portfolio management grew out of the need to improve the financial performance of the large corporate loan portfolios in commercial banks. Lending is the backbone of commercial banking, so lending is what banks should do best. Yet these portfolios proved to be the source of recurring problems and the cause of failure for many institutions. Credit policies, procedures, systems and controls do not always assure asset quality and earnings will be maintained at acceptable levels. A quantitative approach is necessary for effective loan portfolio management.

Objectives of measuring credit risk of a bank’s corporate loan portfolio is to design a capital adequacy framework that responds dynamically to changes in credit quality, alerts bank management, supervisors, and others to emerging problems more quickly than under current capital accord, better suited to the complex activities of large, capable of adapting to market and product evolution, encourages banks to invest additional resources in risk management activities.

Corporate credit risk models split into structural and reduced form modeling and these are reviewed in more detail in next Section. Structural models were introduced by Merton (Merton, 1974), while reduced form models were developed from the work of Jarrow and Turnbull (Jarrow, Lando, & Turnbull, 1995).

Through effective management of credit risk exposure, banks not only support the viability and profitability of their own business but also contribute to systemic stability and to an efficient allocation of capital in the economy.

As mentioned before, in Section 2 we review different models for estimating the credit risk of portfolios of corporate loans, in particular the structural and reduced form models. In section 3 we explain the empirical data and methodology employed for simulating historical
observations, estimating future sectoral default rates and calculating bank’s capital requirement. Section 4 describes the data and discusses the empirical findings, we model the VaR of portfolio to optimize it in this section and finally section 5 covers the implications and conclusions.

2. Literature Review

Corporate bonds are the way lending to large companies is structured and losses occur with these (a) because the company defaults; (b) because the rating of the loan is lowered by rating agencies, like Standard and Poor or Moodys, and hence the loan is worth less in the market because of the increased risk of default; (c) the difference between the value of a bond of a given rating and that of the highest grade bond increases. This last problem affects the whole bond market and so is considered to be a market risk rather than a credit risk. The first split in models is between those which only try to estimate the chance of default, called default mode (DM) models, and those which try to estimate both the chance of default and of ratings changed, called mark-to-market (MTM) models. The second split is between structural models which relate the value of a bond and its default probability to the capital and debt structure of the firm, and the reduced form models, which model the default process and/or the rating change process directly using historical estimates. Lastly there is split between static models which can only model the credit over a fixed time interval and dynamic models which allow one to model over any time interval. (Thomas, 2009)

A variety of analytical techniques have been used for credit risk assessment. They include statistical methods, models based on contingent claims and asset value coverage of debt obligations, operational research (OR) such as linear or quadratic programming, data envelopment analysis (DEA) and neural networks models.

Since the pioneering work of Altman (Altman, 1968) a plethora of statistical bankruptcy prediction studies have been done using multivariate discriminant analysis (Deakin, 1972) (Blum, 1974), logistic regression (Martin, 1977) (Ohlson, 1980) (Zavgren, 1985) (Keasey, McGuinness, & Short, 1990), and probit analysis (Zmijewski, 1984) (Skogsvik, 1990). More recent works include Kolari who developed an early-warning system for bank failure based on logit analysis and trait recognition (Kolari, Glennon, Shin, & Caputo, 2002). Jones and Hensher used a mixed logit model to predict financial distress (Jones & Hensher, 2004), and Canbas who combined discriminant analysis, probit, logit and principal component models to create an integrated early-warning system for bank failure (Canbas, Cabuk, & Kilic, 2005). The reduced approach developed from the idea of Pye (Pye, 1974) that one could use an actuarial approach and model the default rate of a group of bonds over time as is done in mortality tables for humans. This is a static, one-period or infinite period model if one thinks of the one period being infinitely repeated). Jarrow and Turnbull were able to reach out the idea to a dynamic model by replacing the fixed probability of default by a hazard rate which depicts the changes in the default probability over the duration of the loans. (Jarrow & Turnbull, 1995) The basic approach is a DM model but by modelling the dynamics of the process in terms of a Markov chain model on the ratings grades including a defaulted ratings grade, they were about to develop a MTM version of this approach. (Jarrow, Lando, & Turnbull, 1995)

As Guo et al (Guo, Jarrow, & Zeng, 2005) indicate structural models are based on the information on asset values and liabilities available only to the firm’s management while reduced form models are based on information available in the market, that there is only indirect information on a firm’s assets and liabilities. Reconciliation models are being developed which do not have complete information on the dynamics of the process that
triggers default in the structural models and hence lead to cumulative rates of default as in reduced form models. (Cuny & Lejeune, 2003)

Finally there are the static scorecard models which search for estimate the probability of default in a fixed period of time as a function of accounting ratios of firms, and now including other market information. These are DM models. (Thomas, 2009). DM models are the static scorecard models which seek to estimate the probability of default as a function. It was pioneered by Altman. (Altman, 1968)

This empirical research attempts to measure the credit risk of a sample bank’s corporate loan portfolio using advanced IRB approach. The foundation of this method is based on assigning a credit rating to each individual loan and individual firm considering both historical or expected default rates and cross-sectoral correlations. Historical observations for each type of loan and firm must be sufficiently large to have a nicely shaped probability distribution. If not, the observed default rates may be simulated a sufficient number of times to obtain a smooth probability distribution. Finally we model the default rate of portfolio and optimize it in order to minimize the risk of portfolio as well as satisfying Constraints.

3. Data and methodology

We used data involving installment loans of corporates. These data were supplied by a major commercial bank in Iran. The data included all the payment behavior registered for these corporates from March 2007 up to March 2009 covering a total of 36 months. Table 1 shows the name of the sectors and the numbers assigned to.

<table>
<thead>
<tr>
<th>Sector Name</th>
<th>Sector Number Assigned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>1</td>
</tr>
<tr>
<td>Exports</td>
<td>2</td>
</tr>
<tr>
<td>Construction and housing</td>
<td>3</td>
</tr>
<tr>
<td>Manufacturing and mining</td>
<td>4</td>
</tr>
<tr>
<td>Trade services and miscellaneous</td>
<td>5</td>
</tr>
</tbody>
</table>

A sector of the portfolio can be viewed as a group of expositions, and its default rate in a specific period can be obtained by calculating the weighted average of the default rate of the expositions in that period, using the exposition amounts for weighting. Using this process, we constructed time loss series for each of the 5 sectors that the bank assigns its loans.

The monthly default rate for each corporate is computed as:

\[ L_i = 1 - \frac{H_i}{A_i + H_i} \]

Where:
- \( L_i \) = Default rate of month \( i \)
- \( H_i \) = The amount of exposition \( i \)
And $A_i$ is the penalty of exposition $i$ and computed by:

$$A_i = \frac{\sum_{j=1}^{m} a_j \times (R_1 + R_2) \times t_j}{365}$$

Where:
- $a_i$ = The amount of exposition $i$ paid $t_j$ days after due date.
- $R_1$ = The interest rate of bank
- $R_2$ = The penalty rate
- $t_j$ = Number of days after due date of exposition.

And the monthly sectoral default rates in each sector are computed as:

$$M_i = \frac{\sum_{j=1}^{n} H_i \times L_i}{\sum_{j=1}^{n} H_i}$$

Where:
- $n$ = Number of corporates
- $L_i$ = Corporate default rate of month $i$
- $H_i$ = The amount of exposition of month $i$
- $M_i$ = Sectoral default rate of month $i$

We used a random sample of 9000 expositions (each exposition represents one installment of a loan) to estimate monthly default rates for 36 months. Table 2 shows the computed sectoral default rate matrix (36 × 10). Since the numbers of historical observations are not sufficient to obtain a smooth probability distribution, the sectoral default rates need to be simulated. We use Monte Carlo simulation as simulation method. First of all we determine the type of statistical distribution function of defaults using a simulation software package, “Arena”. The results support that the default rate series are fit nicely into lognormal distribution at 93% confidence level. The mean and standard deviation of each sector are computed in table 2. Then sectoral variance-covariance and Cholesky Decomposition matrixes of variance-covariance are computed (Table 3 and 4).

Employing a variance-covariance matrix as an input, Monte Carlo simulation is applied for 1,500 times. Based on the simulated default rates, the mean and the standard deviation of expected sectoral default rates are computed. Then, a credit quality rating scale is fitted into the sectoral default rates distributions. Therefore, credit quality ratings are assigned for each sector based on the expected defaults in the future (Graphs 1-5).

Table 2: Historical Sectoral Default Rates

<table>
<thead>
<tr>
<th>Month-year</th>
<th>SECTORS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Apr-2006</td>
<td>0.2542</td>
</tr>
<tr>
<td>May-2006</td>
<td>0.2523</td>
</tr>
<tr>
<td>Jun-2006</td>
<td>0.2439</td>
</tr>
<tr>
<td>Jul-2006</td>
<td>0.2642</td>
</tr>
<tr>
<td>Aug-2006</td>
<td>0.2447</td>
</tr>
<tr>
<td>Sep-2006</td>
<td>0.2469</td>
</tr>
<tr>
<td>Oct-2006</td>
<td>0.2415</td>
</tr>
<tr>
<td>Nov-2006</td>
<td>0.2545</td>
</tr>
<tr>
<td>Dec-2006</td>
<td>0.2474</td>
</tr>
<tr>
<td>Jan-2007</td>
<td>0.2439</td>
</tr>
<tr>
<td>Sector Number</td>
<td>1</td>
</tr>
<tr>
<td>---------------</td>
<td>-------</td>
</tr>
<tr>
<td>1</td>
<td>0.00014</td>
</tr>
<tr>
<td>2</td>
<td>-0.00008</td>
</tr>
<tr>
<td>3</td>
<td>0.00009</td>
</tr>
<tr>
<td>4</td>
<td>0.00005</td>
</tr>
<tr>
<td>5</td>
<td>-0.00003</td>
</tr>
</tbody>
</table>

Table 3: VARIANCE-COVARIANCE MATRIX
TABLE 4. CHOLESKY DECOMPOSITION MATRIX

<table>
<thead>
<tr>
<th>Sector Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01166</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-0.00653</td>
<td>0.04146</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.00749</td>
<td>-0.0023</td>
<td>0.01922</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.00467</td>
<td>-0.00079</td>
<td>0.0086</td>
<td>0.0309</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>-0.00239</td>
<td>0.00944</td>
<td>0.00051</td>
<td>0.00062</td>
<td>0.01951</td>
</tr>
</tbody>
</table>

4. Empirical results and implications

It appears that the values of Monte Carlo simulated expected sectoral mean default rates and standard deviations are close to the values of actual sectoral mean default rates and standard deviations. For example, the actual mean and standard deviations for sector 1 on Table 2 are 24.3% and 1.18%, respectively. The simulated sectoral mean and standard deviation for sector 1 are 24.3% and 1.15%, respectively. Therefore, it may be argued that the simulations are a good representative of historical observations. The distributions of simulated sectoral default rates are presented in Graphs 1 to 5.

As regards to resulted sectoral default rates, suppose that the Bank has disbursed $30 million in Agriculture sector, we can infer that at 95% confidence level less than $7.8 million will never be paid or will not be paid on time. So the bank should reserve $7.8 million as VaR.

To show a practical application for the simulated sectoral default rates, the Monte Carlo simulation presented the situation where a sample bank formed a corporate loan portfolio in the amount of $100 million which was composed of 5 different sectors with weights: \{30,20,15,25,10\}. The credit risk of the bank’s loan portfolio is computed by multiplying the sectoral weights in the portfolio and expected sectoral default probability distribution. Incorporating the effects of cross-sectoral correlations on defaults rates, the simulated default rates are multiplied by Transposed Cholesky Decomposition matrix (Table 4) to calculate the total credit risk of the loan portfolio (Graph 6).

The results show that the capital requirement for the credit risk of a $100 million corporate loan portfolio is $18.04 million at 99%, and $16.89 million at 95% confidence level (Graph 6). So if bank needs to avoid bankruptcy at a 95% confidence level should reserve $16.98 million in the bank.

In order to assign optimum amount of loan in each sector the portfolio is modeled as bellow:

\[ \text{Min } Z_1 = AX \]
\[ \text{Min } Z_2 = X S X' \]
\[ \text{S.T} \]
\[ \sum_{i=1}^{n} x_i = H \]
\[ x_i \geq 0.2 \times I \quad \forall i = 1,...,n \]

Where:
X_i = The fraction of loan in sector i
S = Variance-covariance matrix
A=Average default rate matrix  
H=Total amount of loan portfolio  
I=The amount of money paid to the bank by the Central Bank of Iran for loaning.  
Assume that the bank has allotted $100 million for loaning, $70 million of which is provided by the central bank, thus:  
H=$100 million  
I=$70 million  
Solving the model with lingo, the result is as bellow:  
\[ X_1 = 14.00000 \]  
\[ X_2 = 14.00000 \]  
\[ X_3 = 20.19588 \]  
\[ X_4 = 14.00000 \]  
\[ X_5 = 37.80412 \]  
Therefore the optimum loan assignment is $14 million for Agriculture, $14 million for Manufacturing and mining, $20.19 million for Construction and housing, $14 million for Exports, $37.80 million for Trade services and miscellaneous.

5. Conclusion

The aim of the negotiations on revising the current Capital Accord (Basel I) was to develop a more risk-sensitive framework for determining capital requirements. By means of Monte-Carlo simulation, we propose a VaR measure for the sample portfolio of loans. We also show how calculating VaR can enable financial institutions to evaluate alternative lending policies on the basis of their implied credit risks and loss rates. This research paper attempts to measure the credit risk and capital requirement of a sample bank’s corporate loan portfolio using advanced IRB approach. Since the amount of historical default rates are limited, the expected sectoral default rates are simulated 1,500 times using Monte Carlo approach to get smooth sectoral probability distributions. The simulated default rates are then applied to a theoretical bank’s corporate loan portfolio in the value of $100 million. The calculation results show that the bank needs to allocate $19.05 million capital for credit risk for its loan portfolio using IRB approach. Finally we model VaR of bank in order to minimize it before lending. We consider all constraints that a commercial bank in Iran might have and solve the model for $100 million of lending. The consequence of this research is a fully-fledged range of solutions to deliver outstanding credit performance, with proven results, which are now available to serve VaR of portfolio. In summary, VaR calculating platform has been conceived, designed, created and activated to achieve objectives and to provide credit grantors that can elevate them to a position of credit excellence with the confidence and assurance of achieving excellent results.
References


Annexure

Graph 1. Simulated Default Probability of Agriculture
Graph 2. Simulated Default Probability of exports
Graph 3. Simulated Default Probability of construction and housing
Graph 4. Simulated Default Probability of manufacturing and mining
Graph 5. Simulated Default Probability of trade services and miscellaneous
Graph 6. Default Distribution of $100 Million Corporate Loan Portfolio